Cook's Matter Field in the Stephenson-Kilmister-Yang Theory of Gravity

Nien-En Tung

May 10, 2012

Outline

- ► Introduction
- Results of Cook's theory
- Summary

- Alternative gravitational theory.
- General relativity (GR)

$$S_{EH} = rac{1}{\kappa} \int_{\Sigma} \sqrt{-g} \ R d^4 x \ .$$

▶ Other Lagrangian densities: \sqrt{R} , $R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$, f(R), etc.

Introduction

- Weyl's Unification: To combine the gravitation with the electromagnetism.
- ► A conformal structure. All physics should not be altered after the measurement unit is changed.
- In his work, he proposed the contracted Riemann tensor $R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$ as a new Lagrangian for the unification.

- ► The equation of motion (EOM) of $R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$: Stephenson (1958), Kilmister and Newman (1961).
- Stephenson showed the EOM:

$$\nabla_{\nu}R^{\nu}_{\ \mu\alpha\beta} = 0. \tag{1}$$

With the use of the second Bianchi identity,

$$\nabla_{\mu}R_{\alpha\beta} - \nabla_{\beta}R_{\alpha\mu} = 0 \tag{2}$$

- ▶ In fact, $\nabla_{\mu}R_{\alpha\beta} \nabla_{\beta}R_{\alpha\mu} = 0$ is the same as Yang's gravitational equation by applying the GL(n)-gauge theory.
- ▶ The SKY equation: $\nabla_{\nu}R^{\nu}_{\ \mu\alpha\beta} = 0$.
- Stephenson, Kilmister and Newman showed the solution of the vacuum Einstein equation satisfies the SKY equation.
- ▶ Although Pavelle found unphysical solution to Eq. (2), Stephenson's equation could be improved by some regulation or restriction. (Thompson 1975)

- ▶ The matter field for the SKY equation.
- Kilmister obtained the gravitational equation with the matters

$$\nabla_{\mu}R_{\alpha\beta} - \nabla_{\beta}R_{\alpha\mu} = I_{\alpha\mu\beta},\tag{3}$$

▶ He characterised that the *I* has 20 independent components with restricting by conservation identities:

$$\nabla_{\beta}I^{\alpha\mu\beta} = 0. \tag{4}$$

▶ He did not describe the explicit form for the current density I.

► Camenzind (1975) wrote down a Yang-Mills field equation for SO(3,1):

$$\nabla_{\nu}R^{\nu}_{\ \mu\alpha\beta} = 8\pi G_{N}\widehat{J}_{\alpha\beta\mu},\tag{5}$$

• he chose the current density \widehat{J} as:

$$\widehat{J}^{\alpha\beta\mu} = \nabla^{\beta} \left(T^{\mu\alpha} - 1/2 g^{\mu\alpha} T^{\lambda}_{\lambda} \right) - \nabla^{\mu} \left(T^{\beta\alpha} - 1/2 g^{\beta\alpha} T^{\lambda}_{\lambda} \right), \tag{6}$$

where the tensor \mathcal{T} is the energy-momentum tensor

► Camenzind's theory is one's inability to draw Eq. (5) from the principle of variation.

 Cook (2008) can derive Camenzind-like equation by the variation method of Palatini

 $S_G = \frac{-1}{16\pi} \int_{\Sigma} \sqrt{-g} \ d^4x \ (R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} + 16\pi J^{\alpha\beta}_{\ \mu} \Gamma^{\mu}_{\ \alpha\beta}), \ \ (7)$

where Σ is a spacetime region. The *J*-term is

$$J_{\alpha\beta}{}^{\mu} = \frac{2G_{N}}{c^{4}} \left(\nabla_{\alpha} \bar{T}^{\mu}_{\beta} - \nabla_{\beta} \bar{T}^{\mu}_{\alpha} \right), \tag{8}$$

where $\bar{T}^\mu_\beta=T_\beta^{\ \mu}-\frac{1}{2}\delta_\beta^{\ \mu}T$ and $T=T_\mu^{\ \mu}$ with the Newton constant G_N .

► The EOM:

$$\nabla_{\nu}R^{\nu}_{\ \mu\alpha\beta} = 4\pi J_{\alpha\beta\mu}.\tag{9}$$

- ► The SKY-Cook theory is analogous in mathematical structure with the Maxwell theory.
- ► Cook views the Christoffel symbol as an gauge potential.
- ► Therefore, the Riemann tensor can be viewed as the field strength.
- ▶ In the Lagrangian density, he coupled the current density *J* with the connection.

- To investigate the Friedmann-Robertson-Walker (FRW) solution in Cook's theory.
- The FRW metric in GR.

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\Omega^{2} \right), \tag{10}$$

▶ The Friedmann equation with scalar curvature k = 0

$$H^2 = \frac{8\pi G_N}{3} \rho,\tag{11a}$$

$$\dot{H} + H^2 = -\frac{4\pi G_N}{3} (\rho + 3p),$$
 (11b)

where $H \equiv \dot{a}/a$ is the Hubble parameter.

▶ Dominant species: Radiation $a(t) \propto t^{1/2}$.; matter $a(t) \propto t^{2/3}$.; vacuum $a(t) \propto e^{\Lambda t}$.

▶ The FRW metric in the SKY-Cook theory.

$$\ddot{H} + 4H\dot{H} = 8\pi G_N \left(\frac{1}{2} \left(\dot{\rho} - \dot{p}\right) + H(\rho + p)\right),\tag{12}$$

one can see that Eq. (12) involves a higher-order derivative.

Matter Dominant Case

$$\frac{\dot{a}^2}{2} - \frac{4\pi G_N M}{3a} + \frac{B}{4a^2} = \frac{Ca^2}{4},\tag{13}$$

where B and C are two integral constants.

▶ When B and C are null, we can show that $a(t) \propto t^{2/3}$.

- Radiation Dominant case
- ▶ The dynamical equation in the radiation case is

$$\nabla_{\nu}R^{\nu}_{i0i} = 0, \tag{14}$$

Þ

$$\frac{\dot{a}^2}{2} + \frac{A_0}{4a^2} = -\frac{k}{2} + B_0 a^2,\tag{15}$$

where A_0 and B_0 are two integral constants.

▶ If the universe is at the late time, then the curvature term and the potential term will not be important. That means:

$$\frac{\dot{a}^2}{2} \simeq B_0 a^2. \tag{16}$$

In this case, $a \simeq C_0 \exp(\sqrt{2B_0}t)$ with a constant C_0 .

- ▶ Th initial condition of the universe. $a(0) = 0, \dot{a} > 0$.
- ▶ The solution of the a(t) is $a = C_0 \exp(\sqrt{2B_0}t)$.

- ▶ In the above calculation, one may note that the current density tensor *J* is null while the universe is filled with the radiation.
- ▶ The radiation solution now belongs to the set of the vacuum solutions of Eq. (9) and therefore the light is excluded from the species of the ordinary matters.

- In this definition of the current density J, we can not distinguish the gravitational effect of the radiation from that of the vacuum energy.
- ► Therefore, such a definition of the current density of the matter seems to be improper.
- ▶ Without the matter field, Eq. (1) or Eq. (2) contains the all solution space of the Einstein equation and shows no serious contradiction to Einstein's formula.
- ▶ The matter field for the SKY equation is an open question.